Mobile Systeme
Grundlagen und Anwendungen standortbezogener Dienste

Location Based Services in the Context of Web 2.0

Department of Informatics - MIN Faculty - University of Hamburg
Lecture Summer Term 2007

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CLDC

NMEA

MIDP

Google Earth

OpenGIS

SQL

KML

Bluetooth

Mash-Ups

Web 2.0

J2ME

Loxodrome

Euclidean Spaces

RDMS

GPS

PostGIS

GPX

Maps

JSR 179

Polar Coordinates

Threads

API

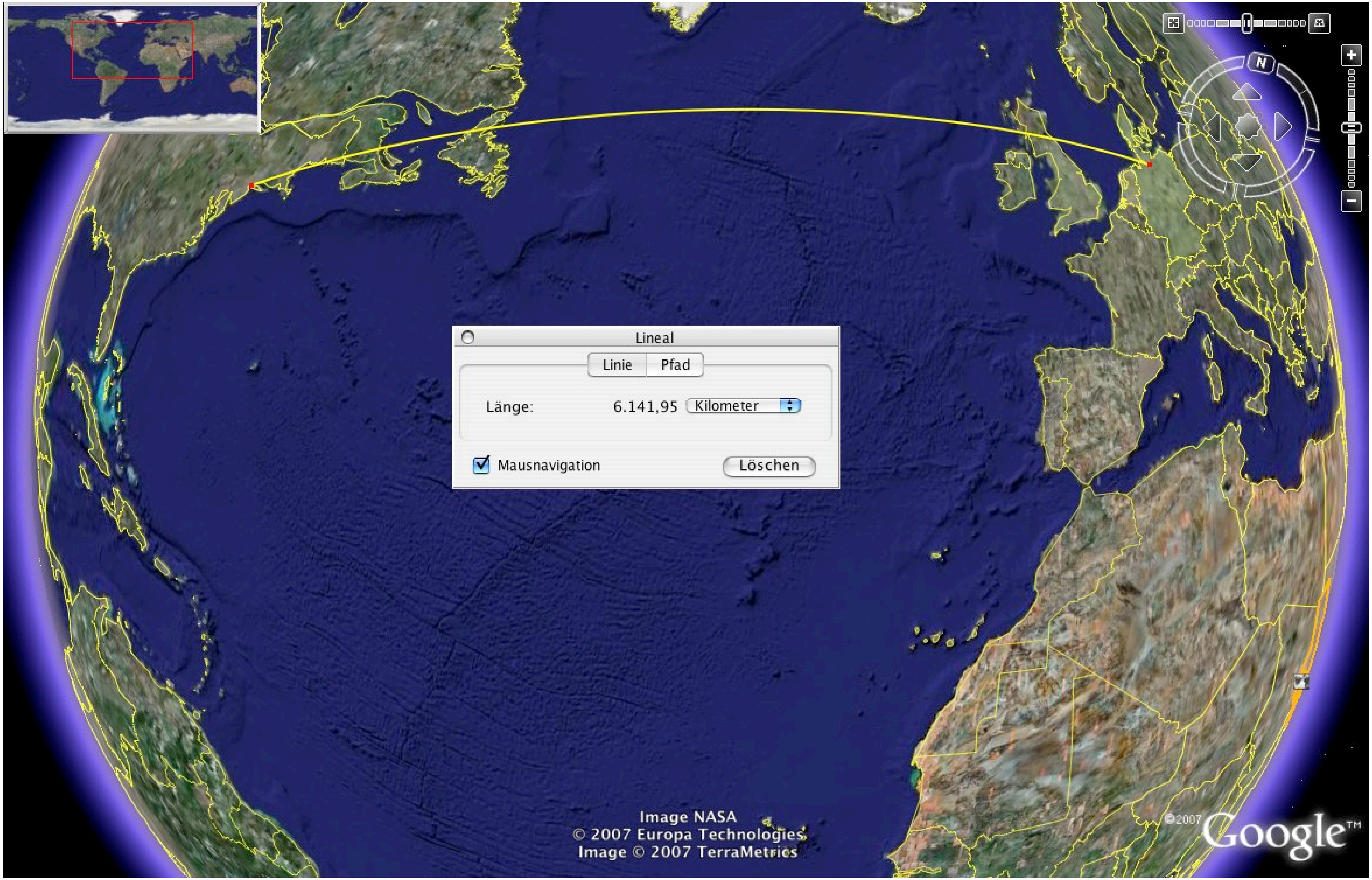
Today: Basics of Geodetics

- Terms and Definitions
- Foundations of Spherical Trigonometry
- Introduction to map projections

Why?

- To design Location Based Systems we need a basic understanding of the geometry of the earth
- What are the restaurants/pubs/stations ... (Point of Interests) nearby?
- Basic questions include
  - How to specify a position on the earth?
  - How can I compute the distance between two given points?
  - How can I compute the direction between two given points?
  - How can I represent the earth surface on a two-dimensional map?

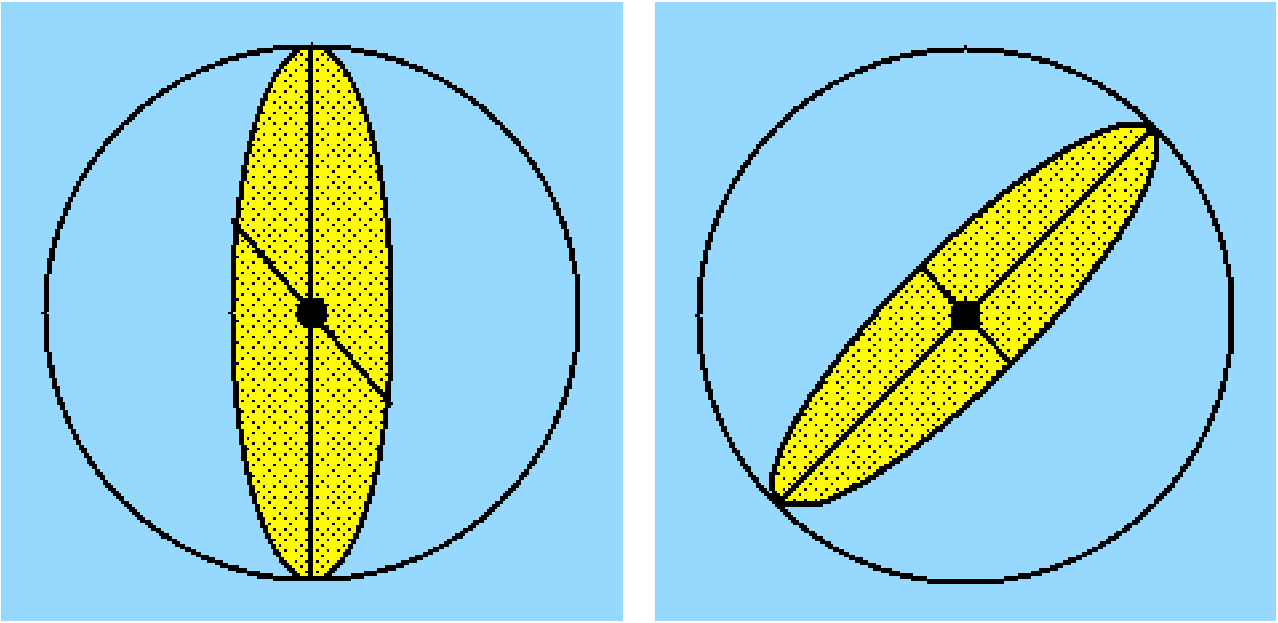
How do they do that?



and...
the earth is a sphere!
(at least in this lecture ;-)

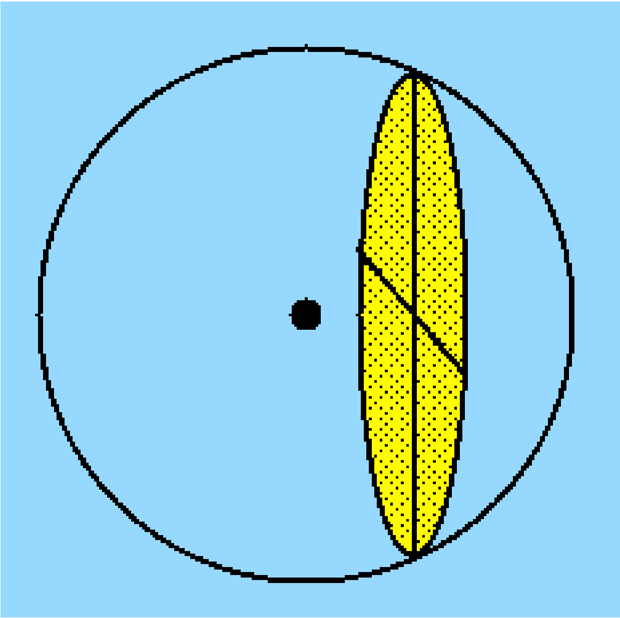
Great Circle

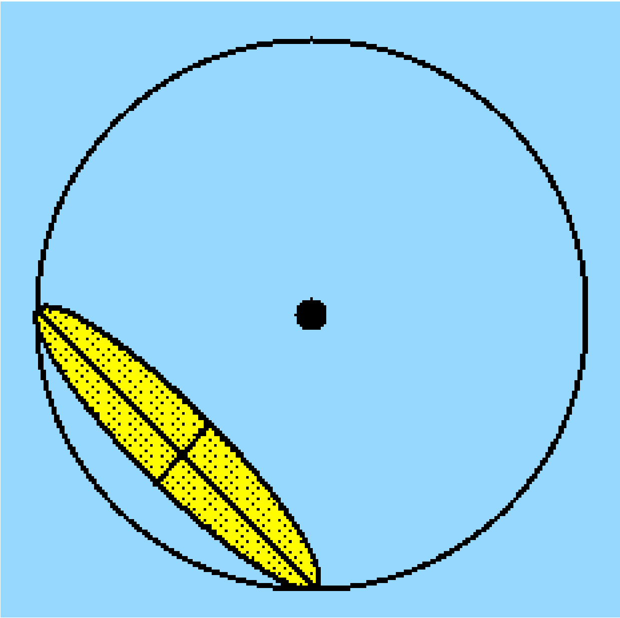
- The shortest path between two points on a plane is a straight line. On the surface of a sphere, however, there are no straight lines. The shortest path between two points on the surface of a sphere is given by the arc of the great circle passing through the two points.
- A great circle is defined to be the intersection with a sphere of a plane containing the center of the sphere.



Small Circle

- If the plane does not contain the center of the sphere, its intersection with the sphere is known as a small circle. In more everyday language, if we take an apple, assume it is a sphere, and cut it in half, we slice through a great circle. If we make a mistake, miss the center and hence cut the apple into two unequal parts, we will have sliced through a small circle.



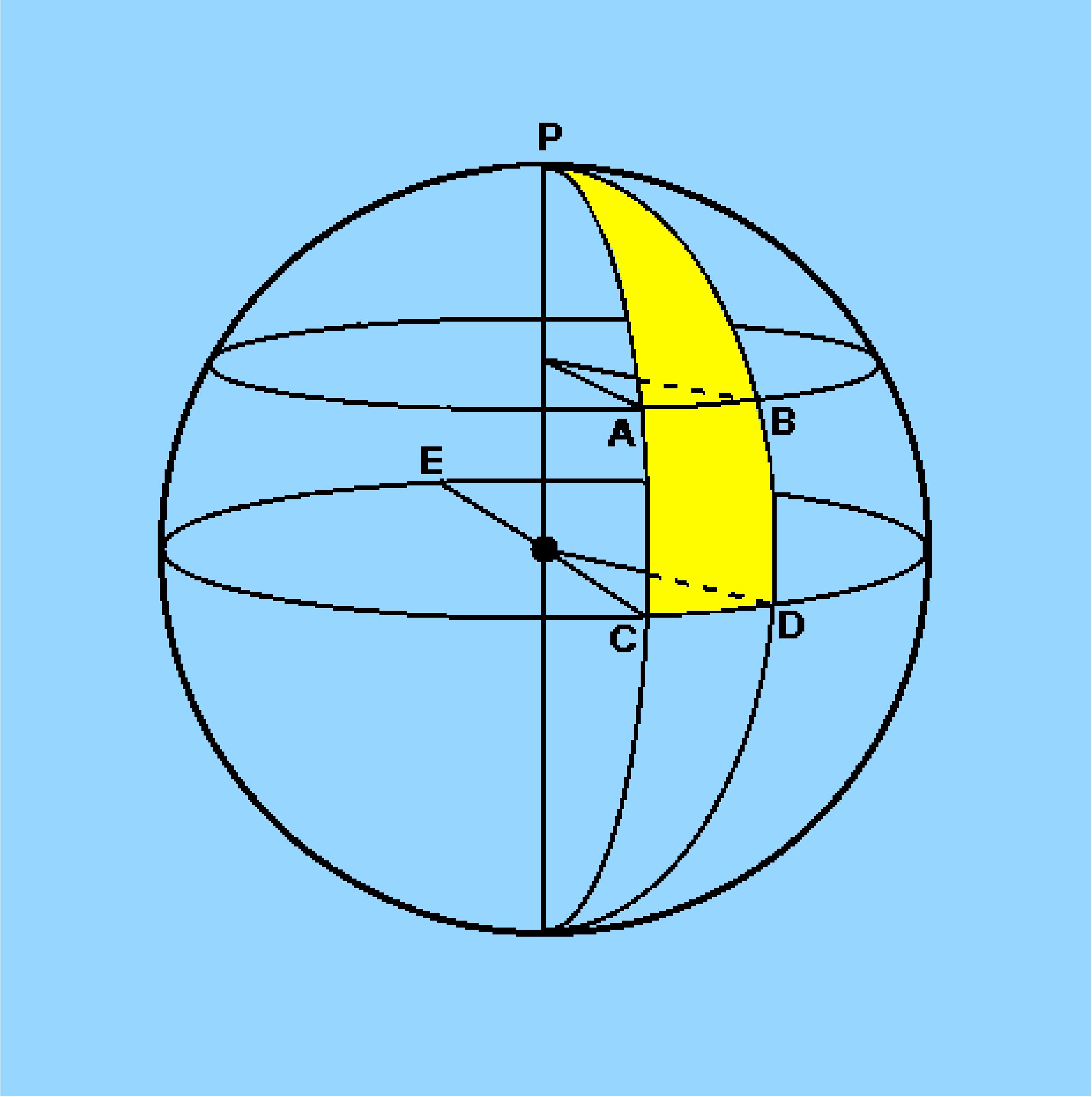


Spherical Triangles (1)

- If we wish to connect three points on a plane using the shortest possible route, we would draw straight lines and hence create a triangle. For a sphere, the shortest distance between two points is a great circle. By analogy, if we wish to connect three points on the surface of a sphere using the shortest possible route, we would draw arcs of great circles and hence create a spherical triangle. To avoid ambiguities, a triangle drawn on the surface of a sphere is only a spherical triangle if it has all of the following properties:
  - The three sides are all arcs of great circles.
  - Any two sides are together greater than the third side.
  - The sum of the three angles is greater than 180°.
  - Each spherical angle is less than 180°.

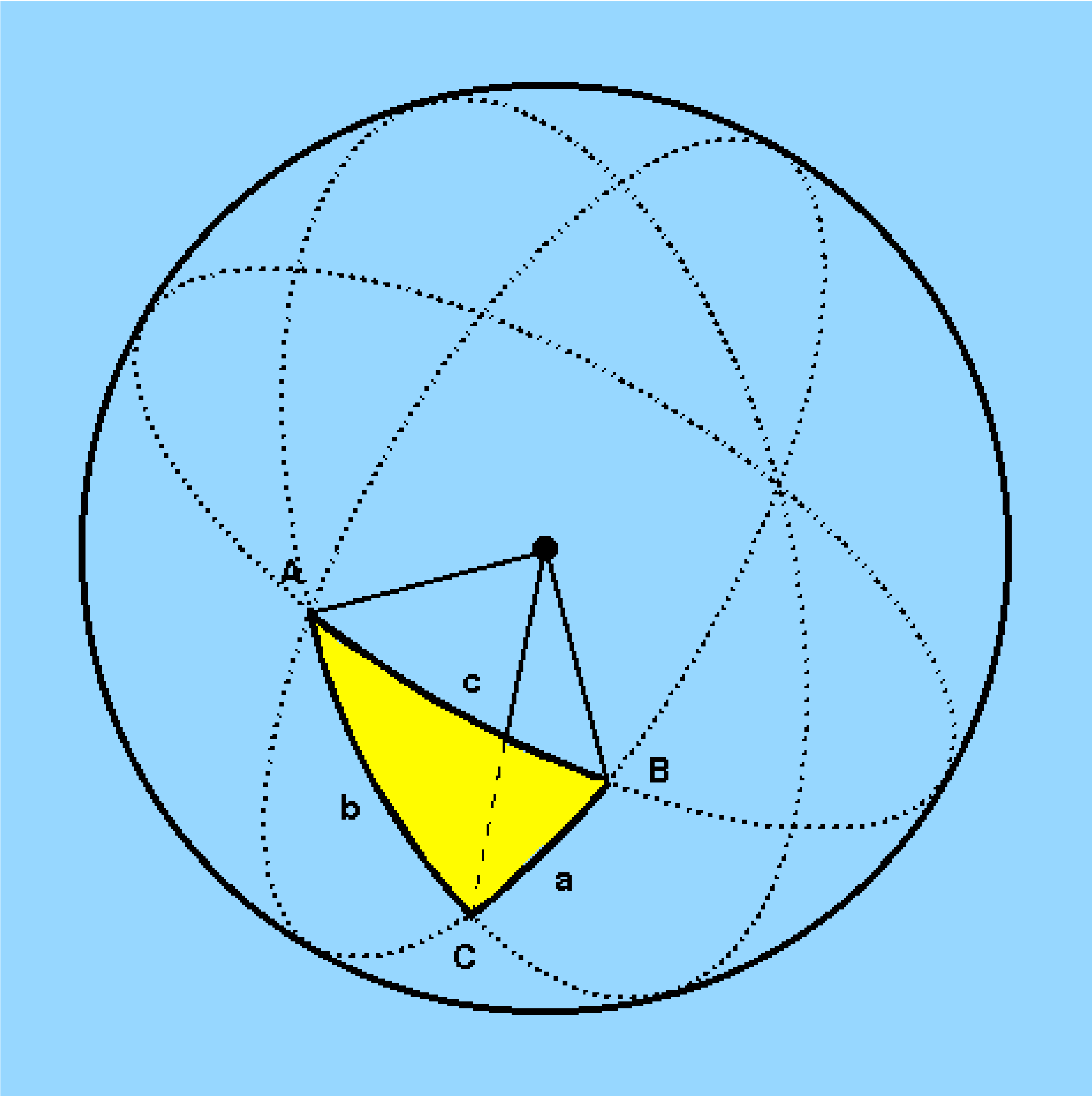
Spherical Triangles (2)

- triangle PAB is not a spherical triangle (as the side AB is an arc of a small circle), but
- triangle PCD is a spherical triangle (as the side CD is an arc of a great circle).



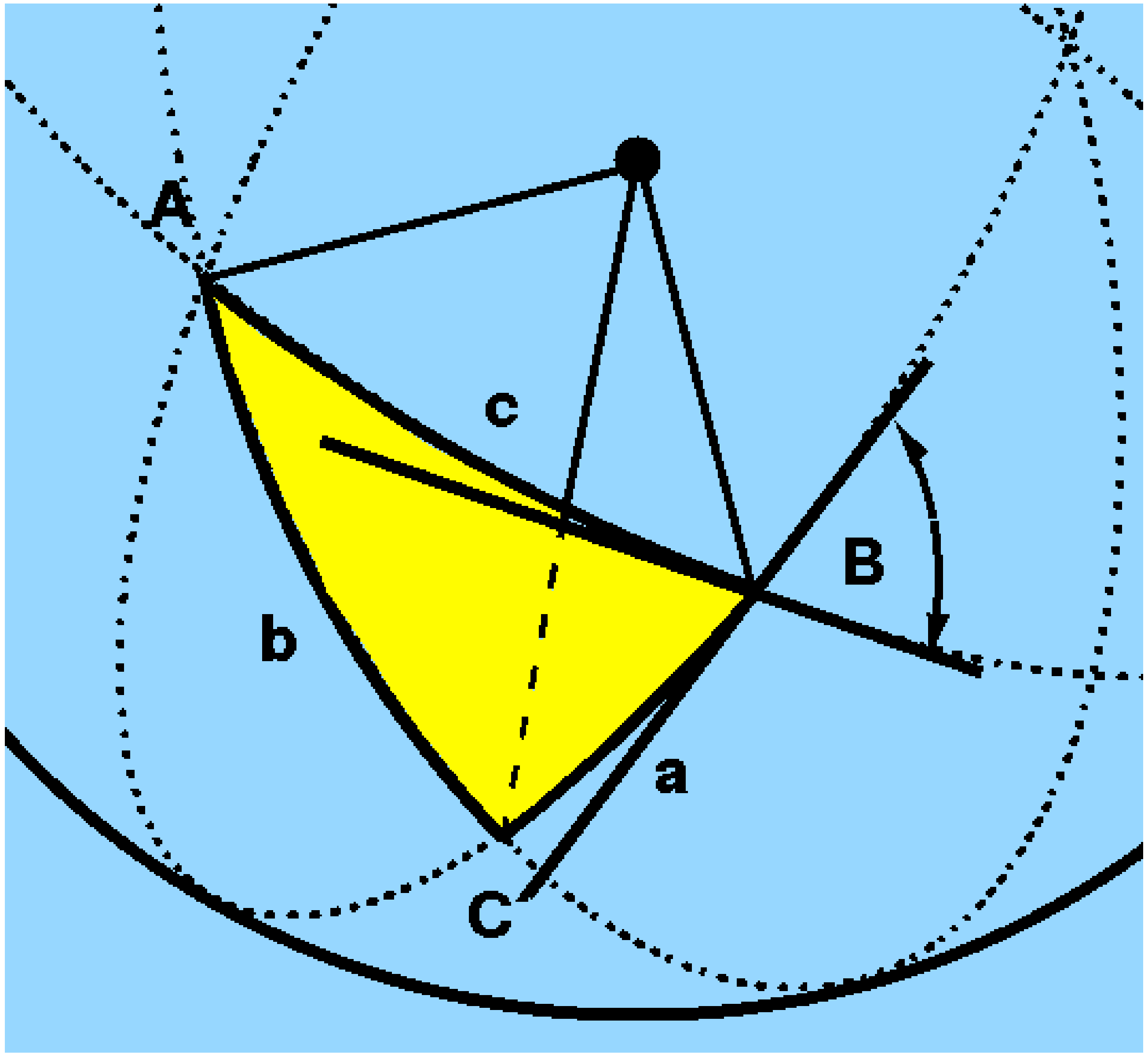
Spherical Triangles (3)

- a spherical triangle, formed by three intersecting great circles, with arcs of length (a,b,c) and vertex angles of (A,B,C).



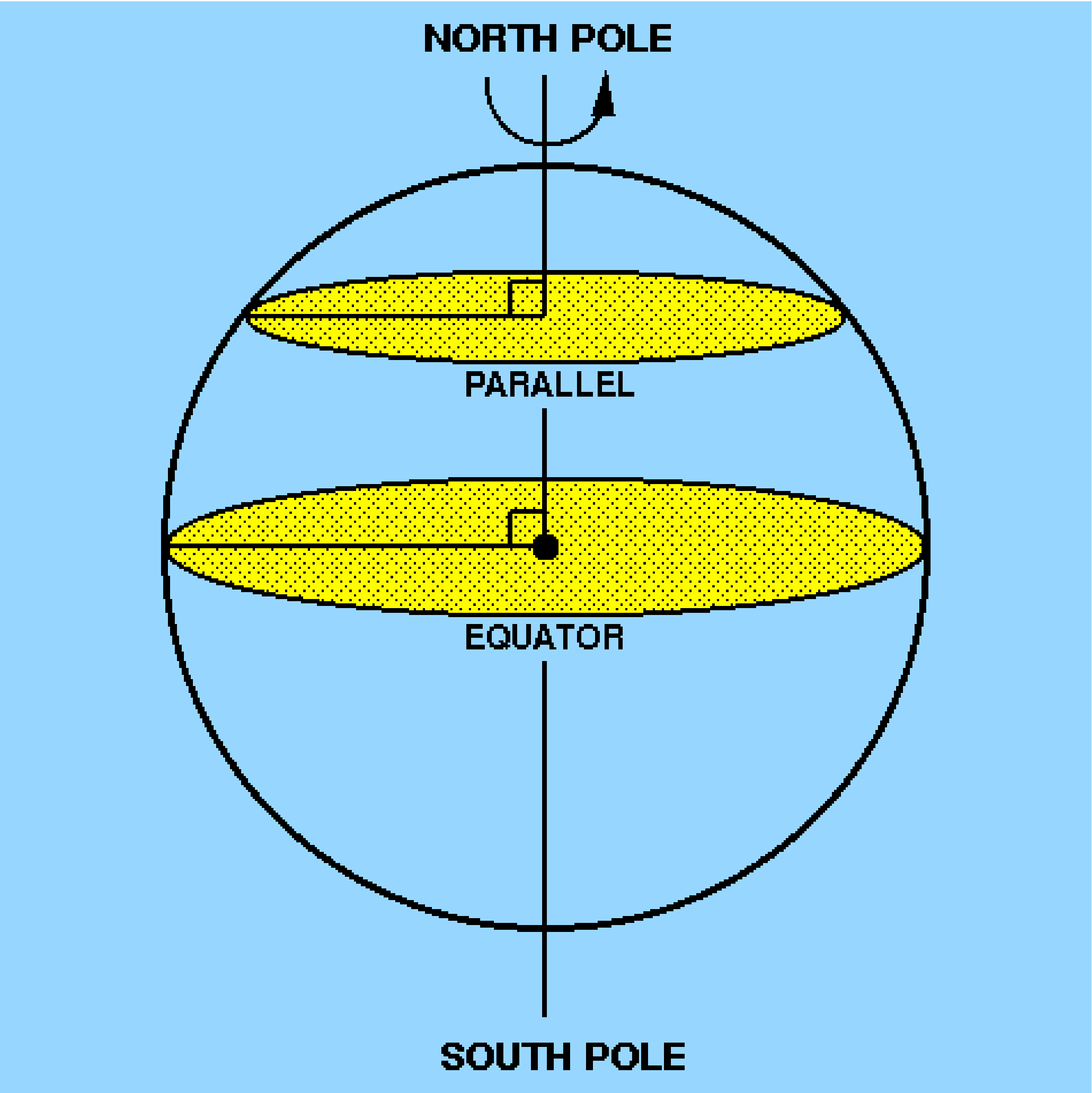
Angles in a Spherical Triangle

- the angle between two sides of a spherical triangle is defined as the angle between the tangents to the two great circle arcs, as shown in the figure for vertex angle B.
- Kugeldreieck Applet



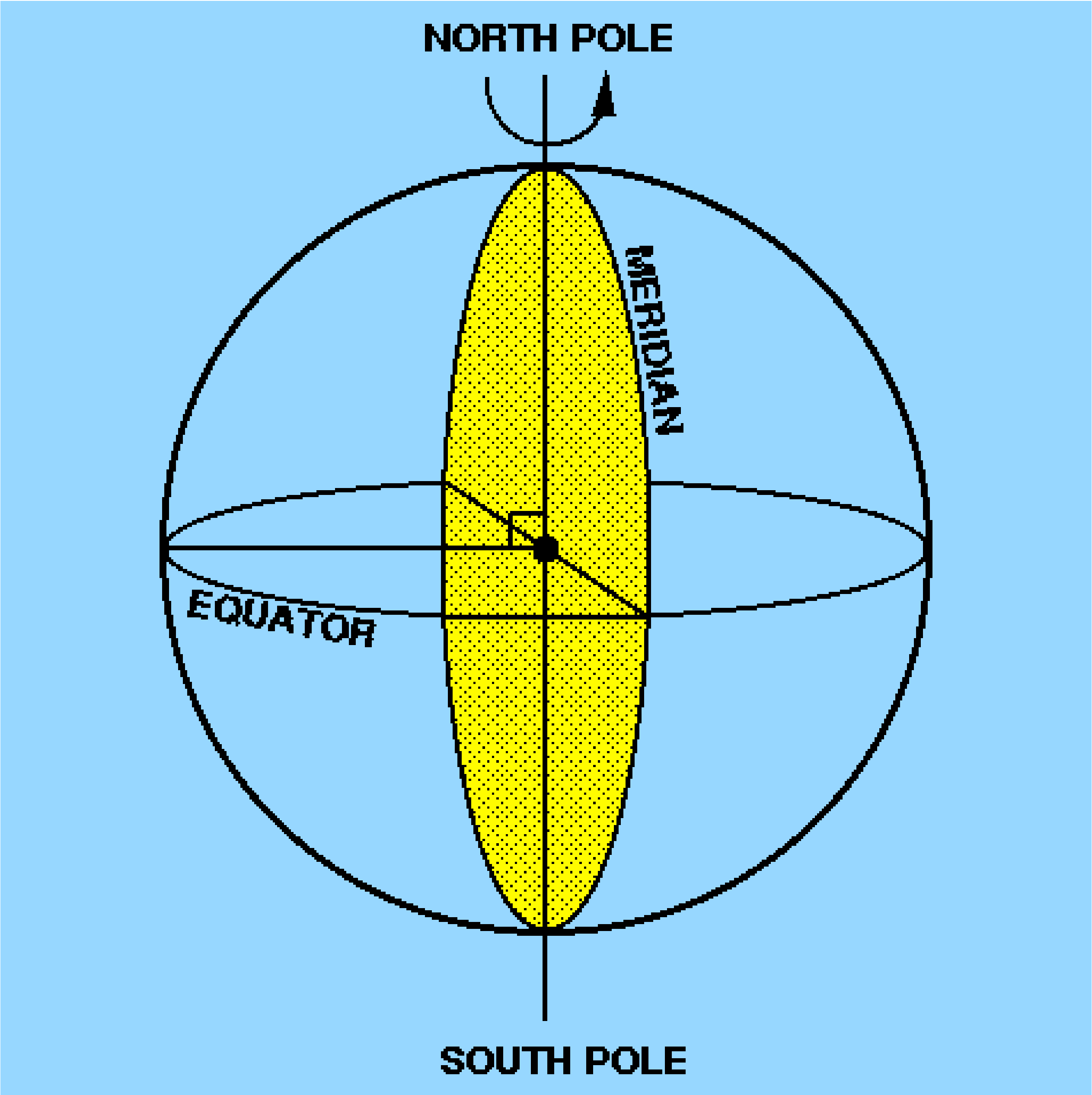
Earth's Surface

- The rotation of the Earth on its axis presents us with an obvious means of defining a coordinate system for the surface of the Earth. The two points where the rotation axis meets the surface of the Earth are known as the north pole and the south pole and the great circle perpendicular to the rotation axis and lying half-way between the poles is known as the equator.



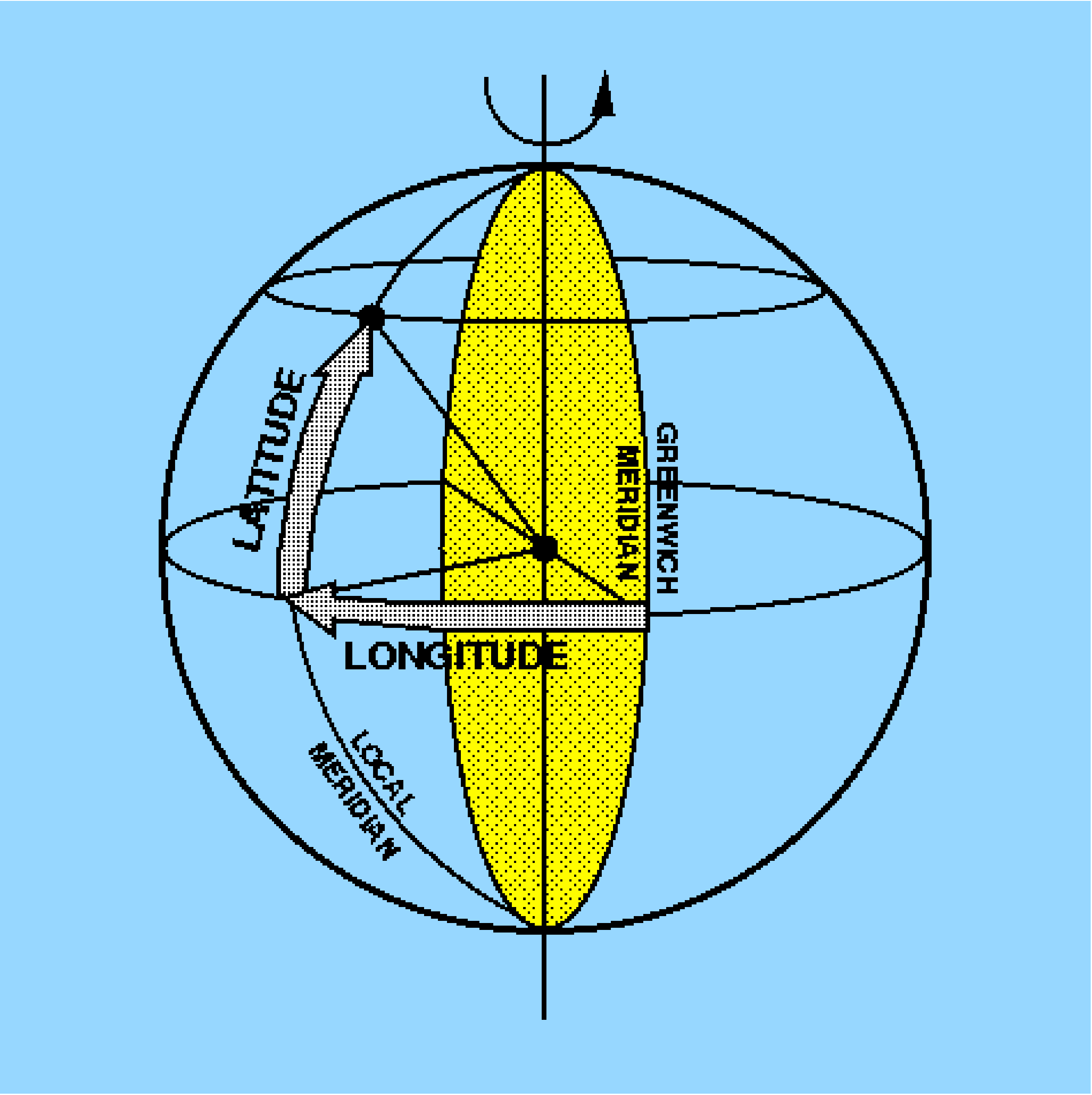
Meridians and latitude lines

- Great circles which pass through the two poles are known as meridians and small circles which lie parallel to the equator are known as parallels or latitude lines.



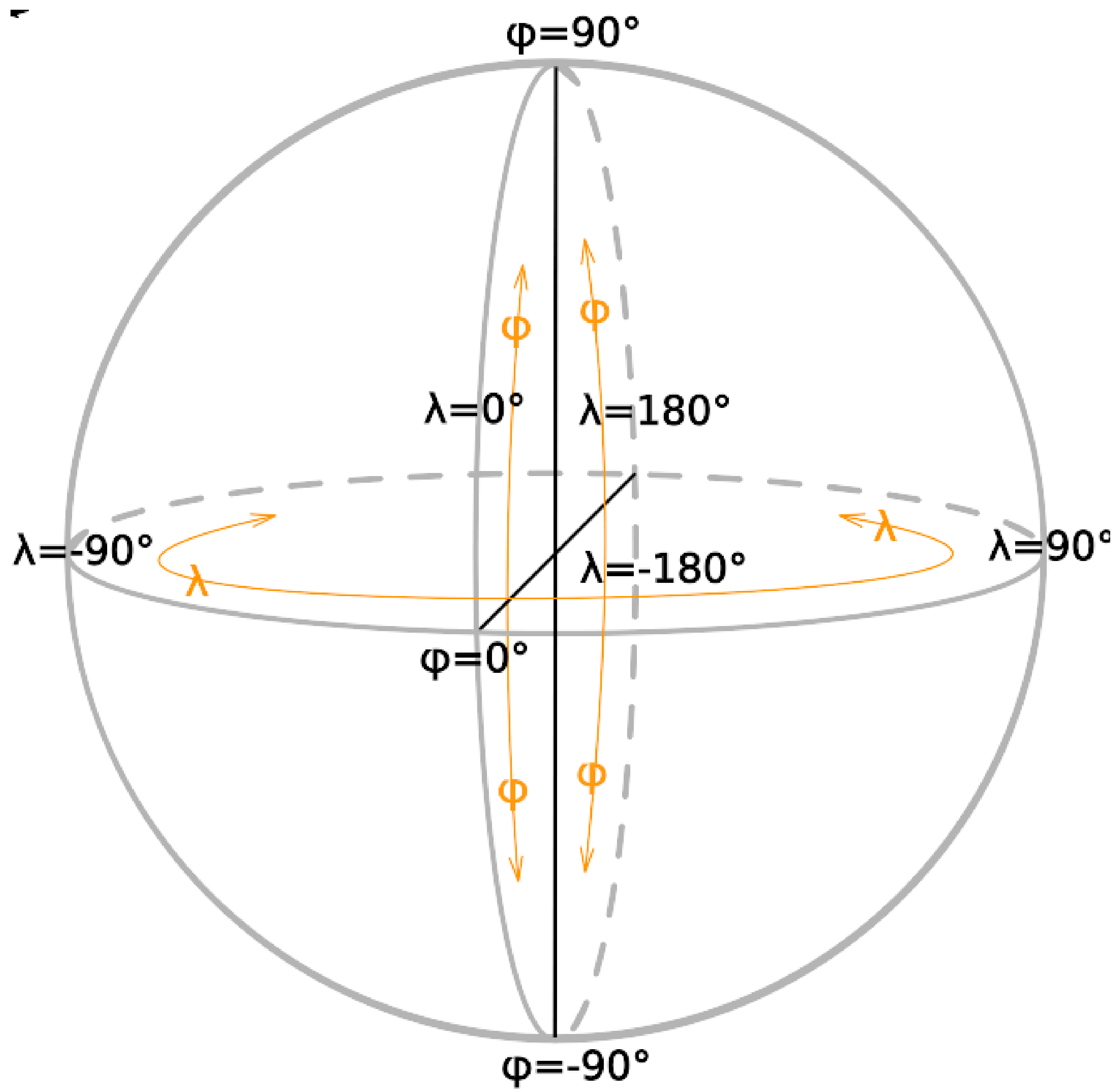
Latitude and Longitude (1)

- The latitude of a point is the angular distance north or south of the equator, measured along the meridian passing through the point.
- Latitude (lat, phi, φ)
  - positive if point is north of the equator
- Longitude (lon, lambda, λ)
  - positive if point is east from Greenwich meridian



Latitude and Longitude (2)

- The latitude φ ranges from 90 degrees (north pole) to -90 degrees (south pole)
- The longitude λ ranges from 180 degrees (east) to -180 degrees (west).
- degrees are divided into minutes (1/60th of a degree, designated by ′ or "m") and seconds (1/60th of a minute, designated by ″ or "s").



Some definitions and formats

- the radius of the earth is about r=6371km. The diameter d=2πr=40030km
- nautical mile is defined as the distance subtending an angle of one minute of arc at the equator
  - 1nm=d/(360*60)=1,85km
  - A speed of one nautical mile per hour is known as one knot
- There are several formats for degrees:
  - DMS Degree:Minute:Second (49°30'00" N)
  - DM Degree:Minute (49°30.0' N)
  - DD Decimal Degree (+49.5000°), generally with 4 decimal numbers.
  - GPS (NMEA) formats positions differently: 4930.00,N

Metric Spaces and cartesian Coordinates

- right-angled coordinate systems play an important role in geo informatics
  - Cartesian coordinate systems are the foundation to visualize Geo Objects in Raster and Bitmaps charts
- In general we define Metric Spaces as reference systems. A metric space consists of a (non empty) Set M and a Metric. A metric is a real-valued function („distance function“) d(a, b) between 2 Elements a and b of M with
  1. $d(a, b) \geq 0$ for all a, b of M, $d(a, b) = 0$ iff $a=b$
  2. $d(a, b) = d(b, a)$ (symmetry)
  3. $d(a, b) \leq d(a, c) + d(c, b)$ (triangle inequality)

Euclidean Metric

- the most important metric in Geo Informatics is the Euclidian Metric:

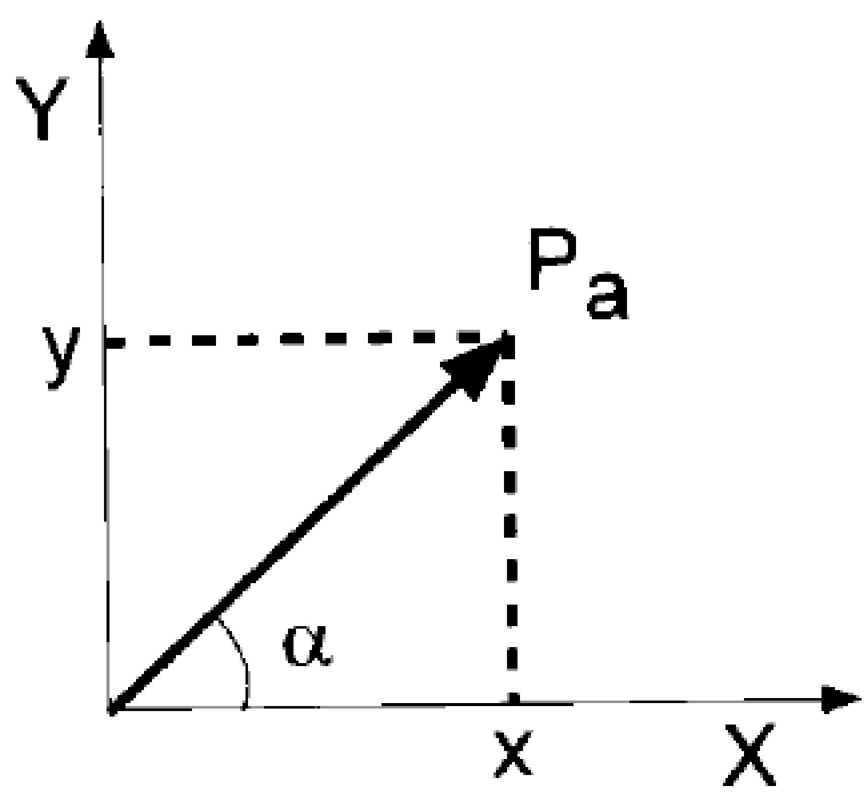
$$d(\mathbf{x},\mathbf{y}) = \|\mathbf{x}-\mathbf{y}\| = \sqrt{\sum_{i=1}^{n}(x_i - y_i)^2}.$$

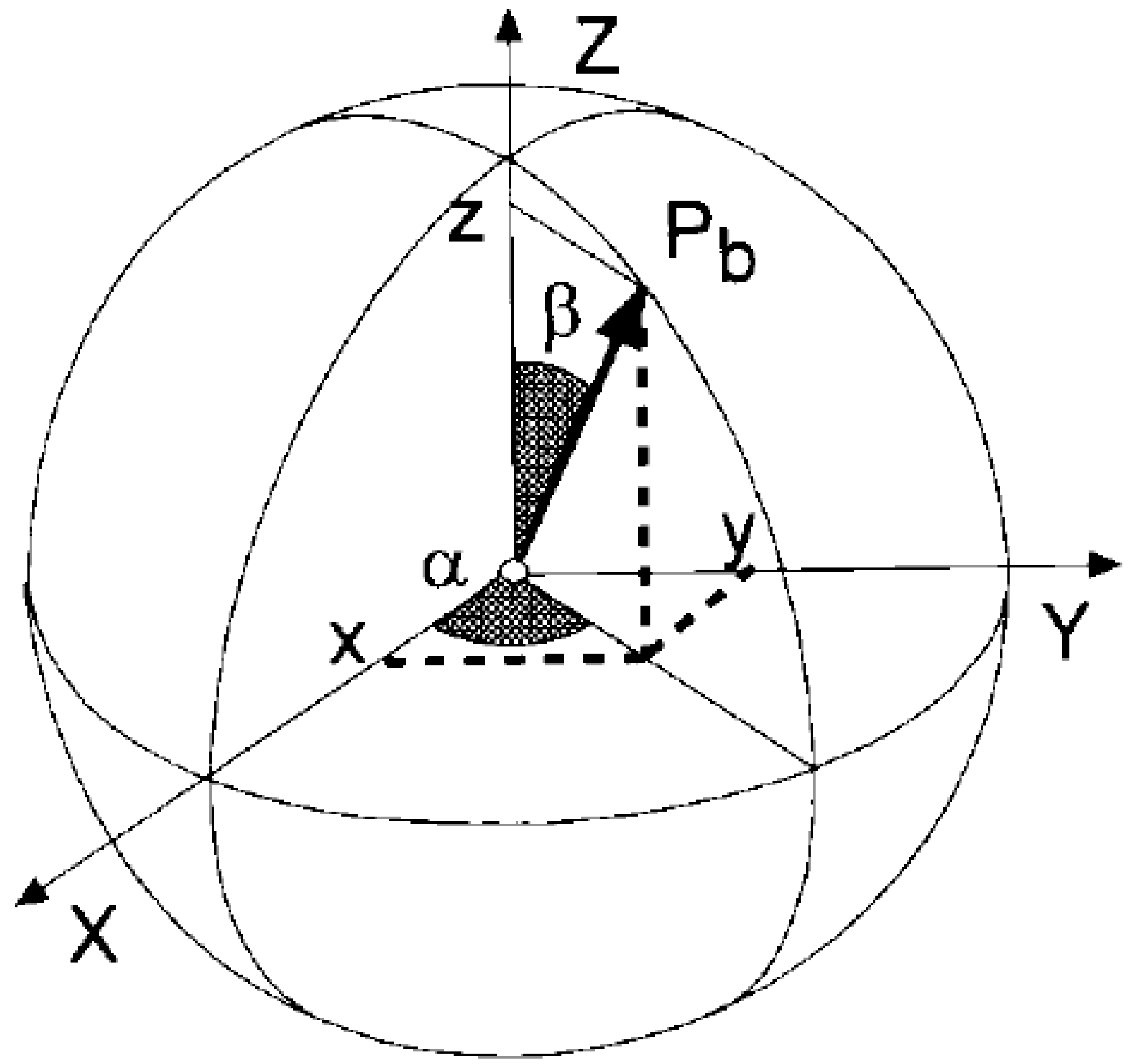
- a Generalization is the Minkowski Metric (L_p distance):

$$L_p(x,y) = \sqrt[p]{\sum_{i=1}^{n}|x_i - y_i|^p}, \quad x,y \in \mathbb{R}^n$$

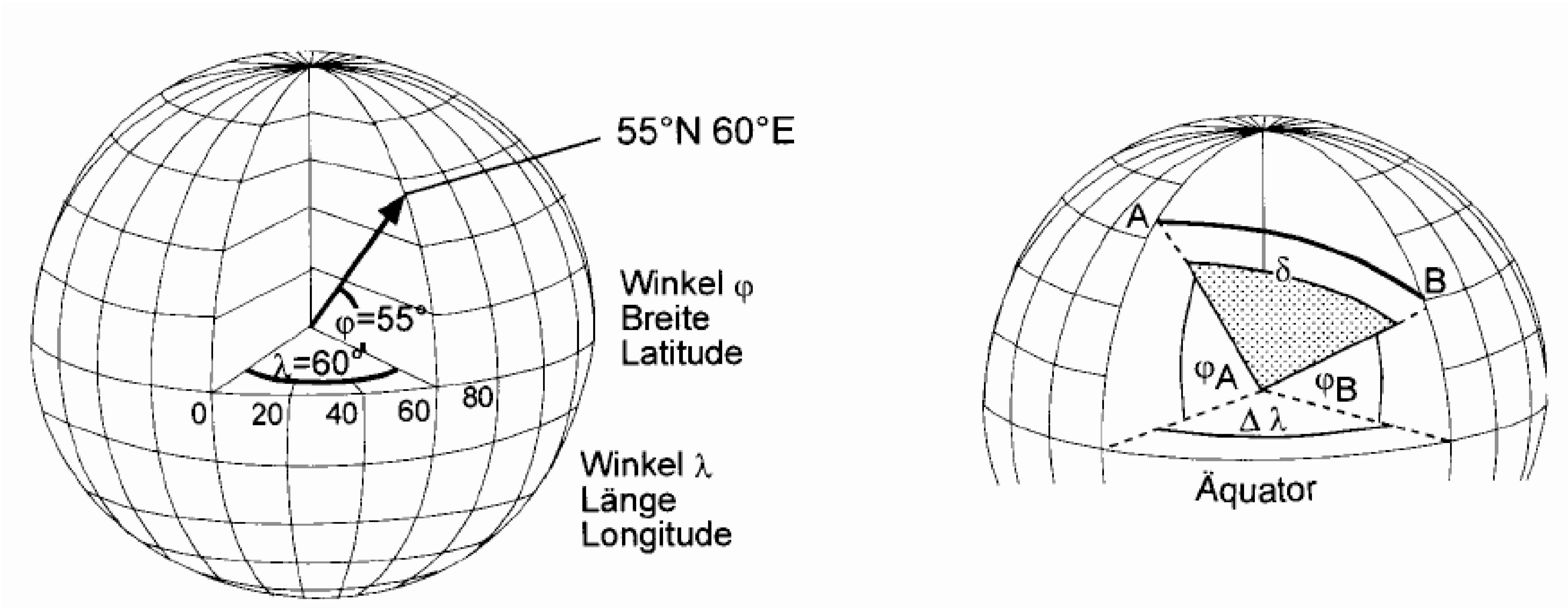
  - with p=2: Euclidean Metric
  - with p=1: City-Block or Manhattan-Metric

Cartesian & Polar Coordinates





Shortest distance of 2 points on earth



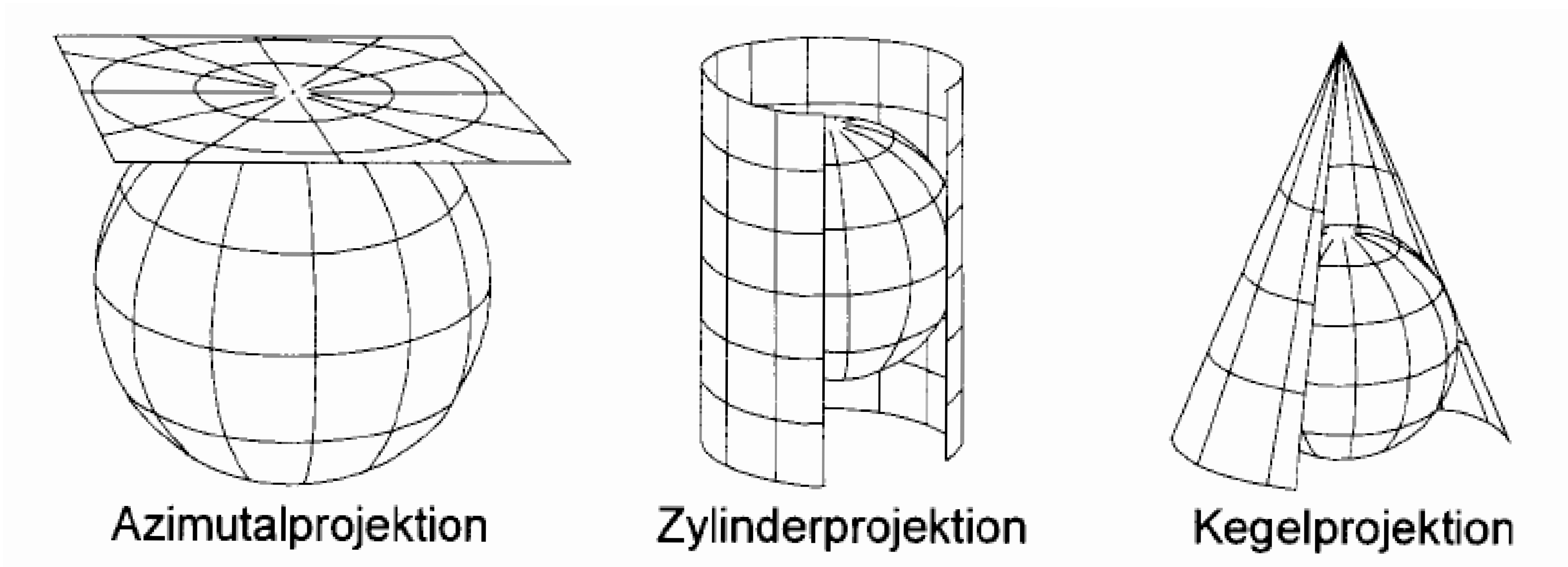
- 2 Points define a Great Circle, the shortest Path is called Orthodrome
- $d(A, B) = R * \cos^{-1} (\sin \varphi_A * \sin \varphi_B + \cos \varphi_A * \cos \varphi_B * \cos(\lambda_A - \lambda_B))$
- cf. http://plus.maths.org/issue7/features/greatcircles/

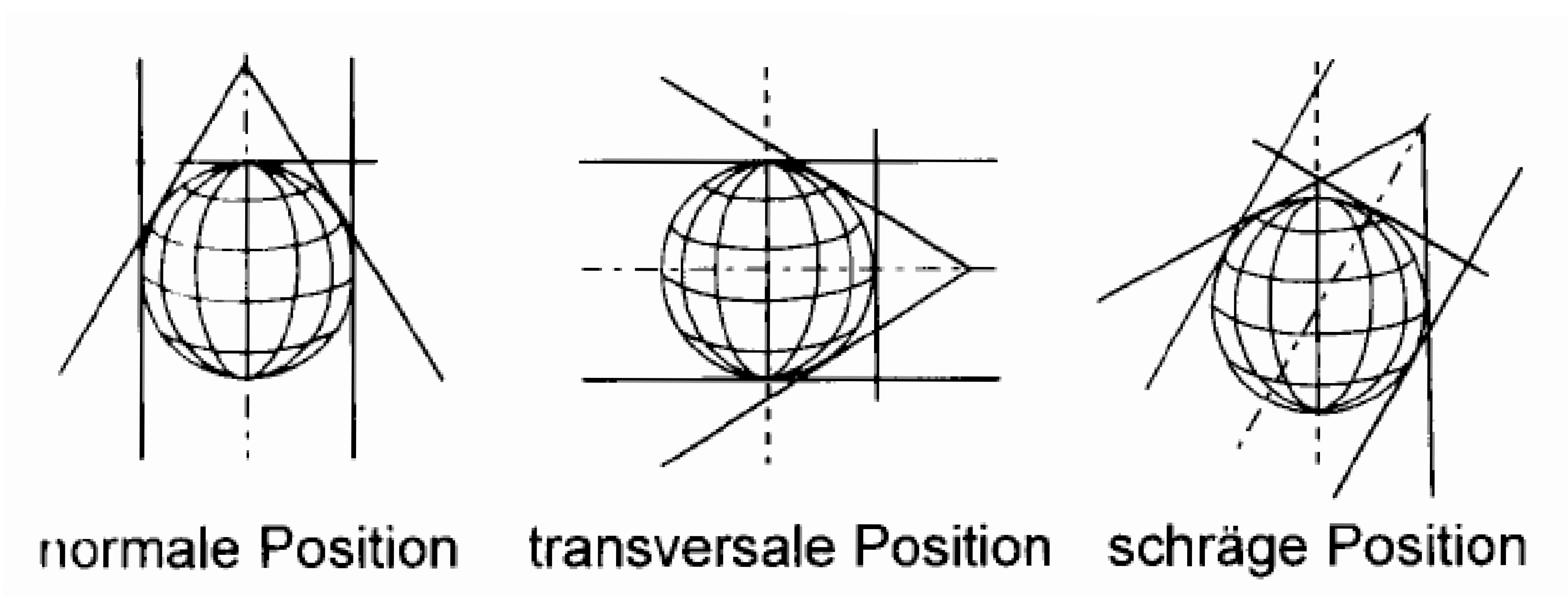
Introduction to map projections

The earth is (almost) a sphere ...

- unfortunately the earth cannot be projected on a (flat) plane easily
  - in other words: a map projection of the earth cannot be equidistant (längentreu, äquidistant), equal-area (flächentreu, äquivalent) and conformal (winkeltreu, konform) at the same time
- equidistant projection: A projection that maintains accurate distances from the center of the projection or along given lines (e.g. equator of (standard) Mercator projection)
- equal-area projection: A flat map so drawn that equal units of actual (or represented) area in any two portions of the map have identical map areas. Equal-area projections can never be conformal.
- conformal projection: A map that preserves angles; that is, a map such that if two curves intersect at a given angle, the images of the two curves on the map also intersect at the same angle.

Map projections

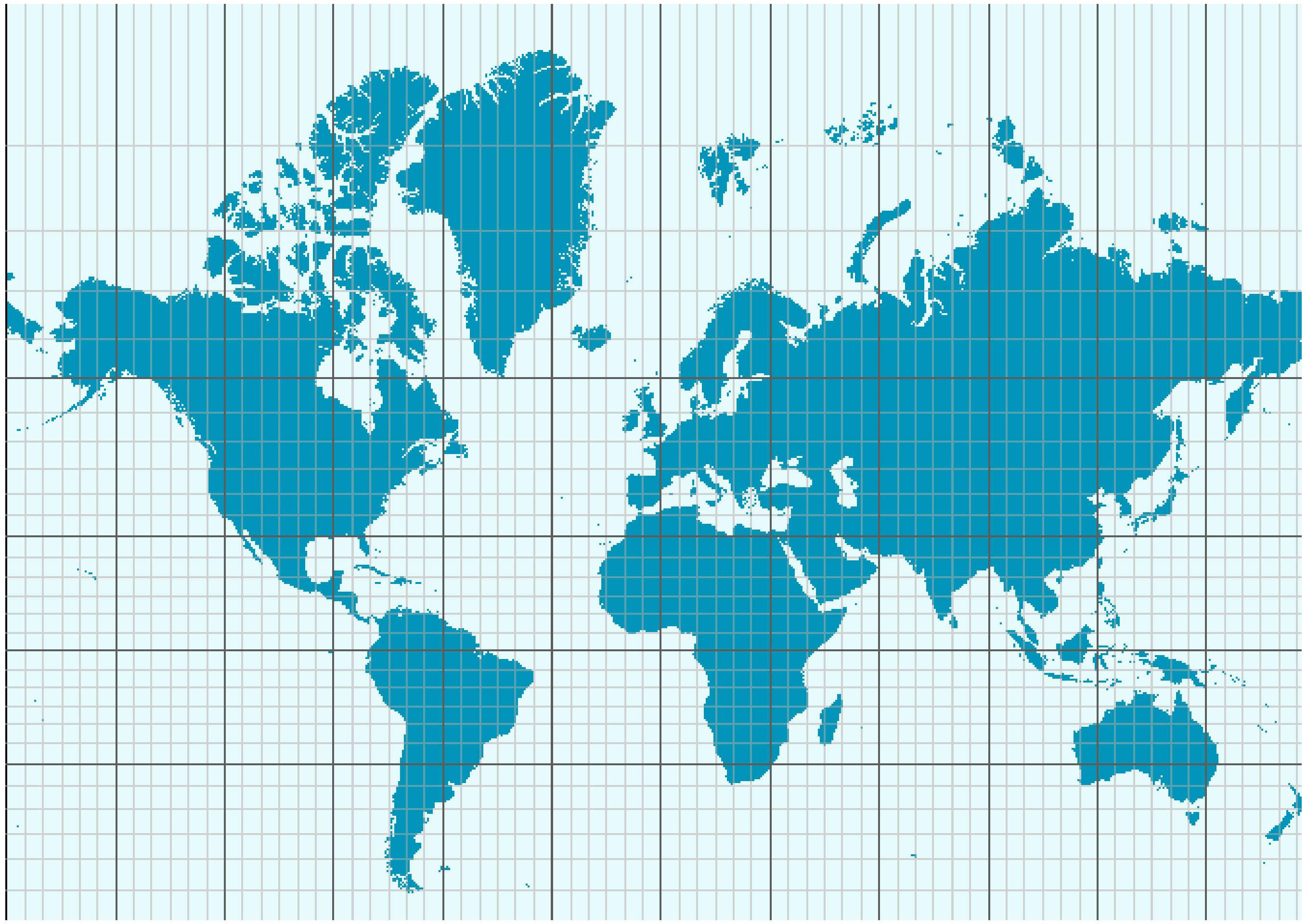




Azimuthal projection

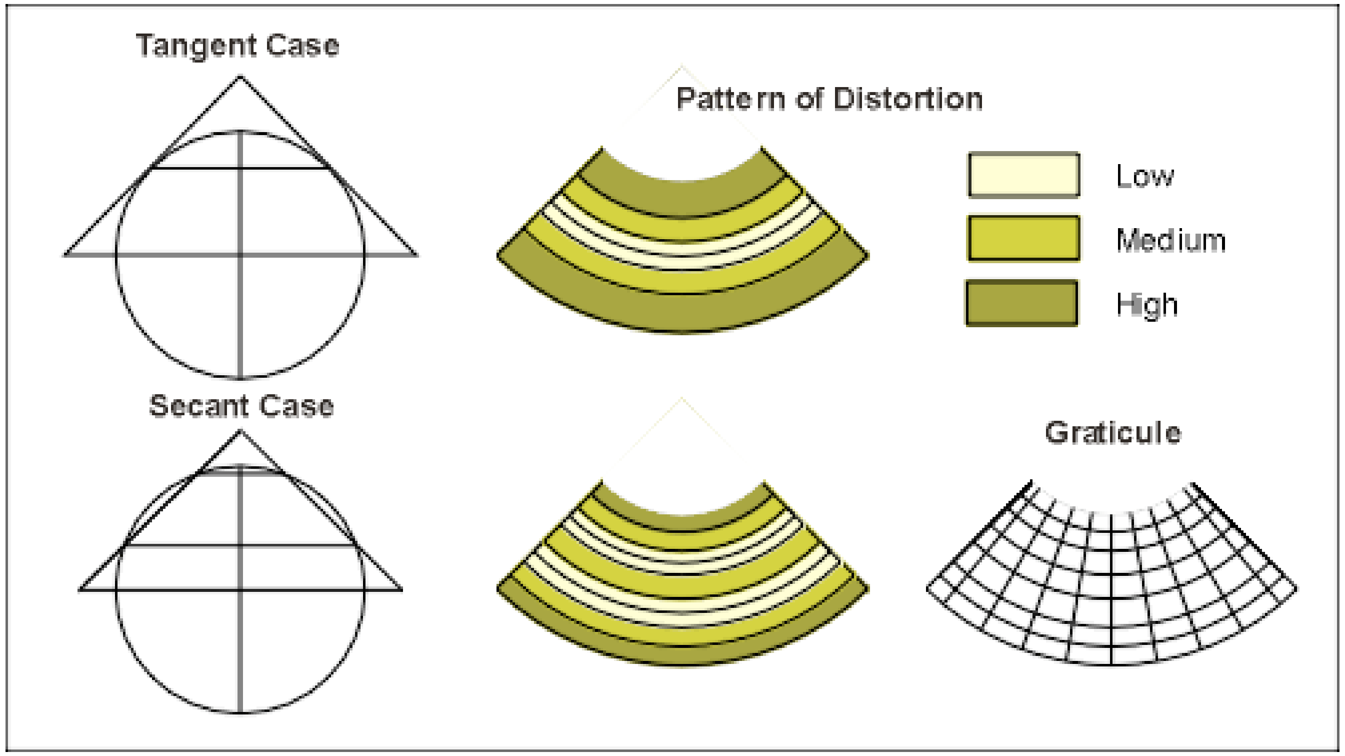
- azimuthal projection in polar, transversal and angular position
- all directions (azimuth) from the reference point to any other point are correct
- all Great Circles through the reference point are straight lines

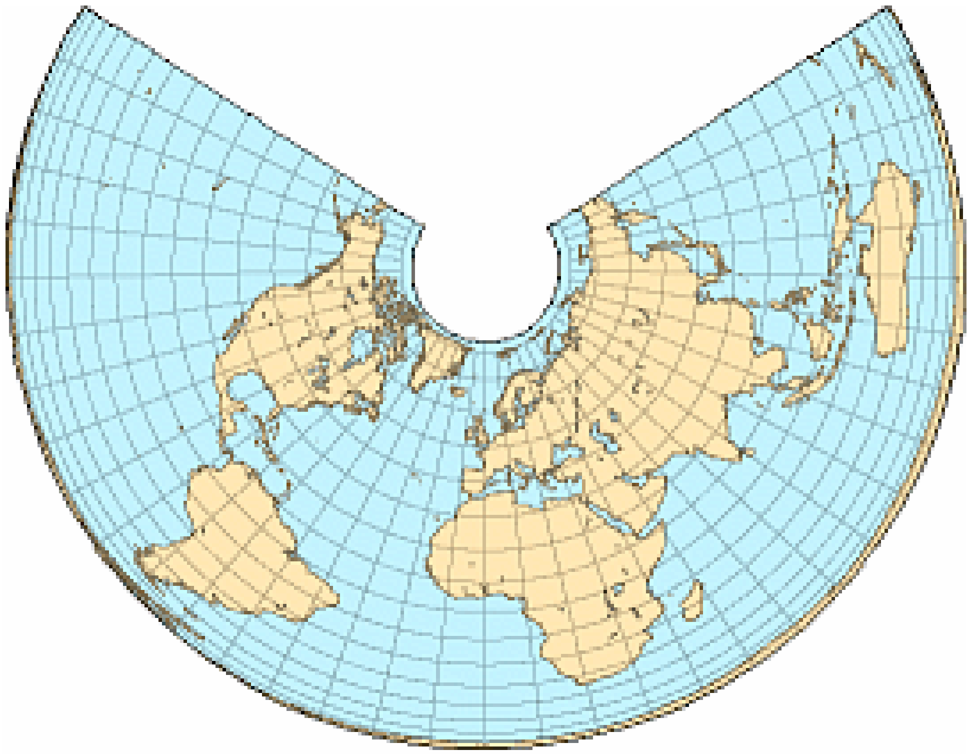
Cylindric projection (e.g Mercator)



- Mercator projection: Axis of earth correspond to axis of cylinder, reference circle is the equator -> equator is mapped equidistantly
  - Map is conformal, i.e. not equal-area
  - Meridians have the same distance in contrast to the distances of parallels

Conical projection

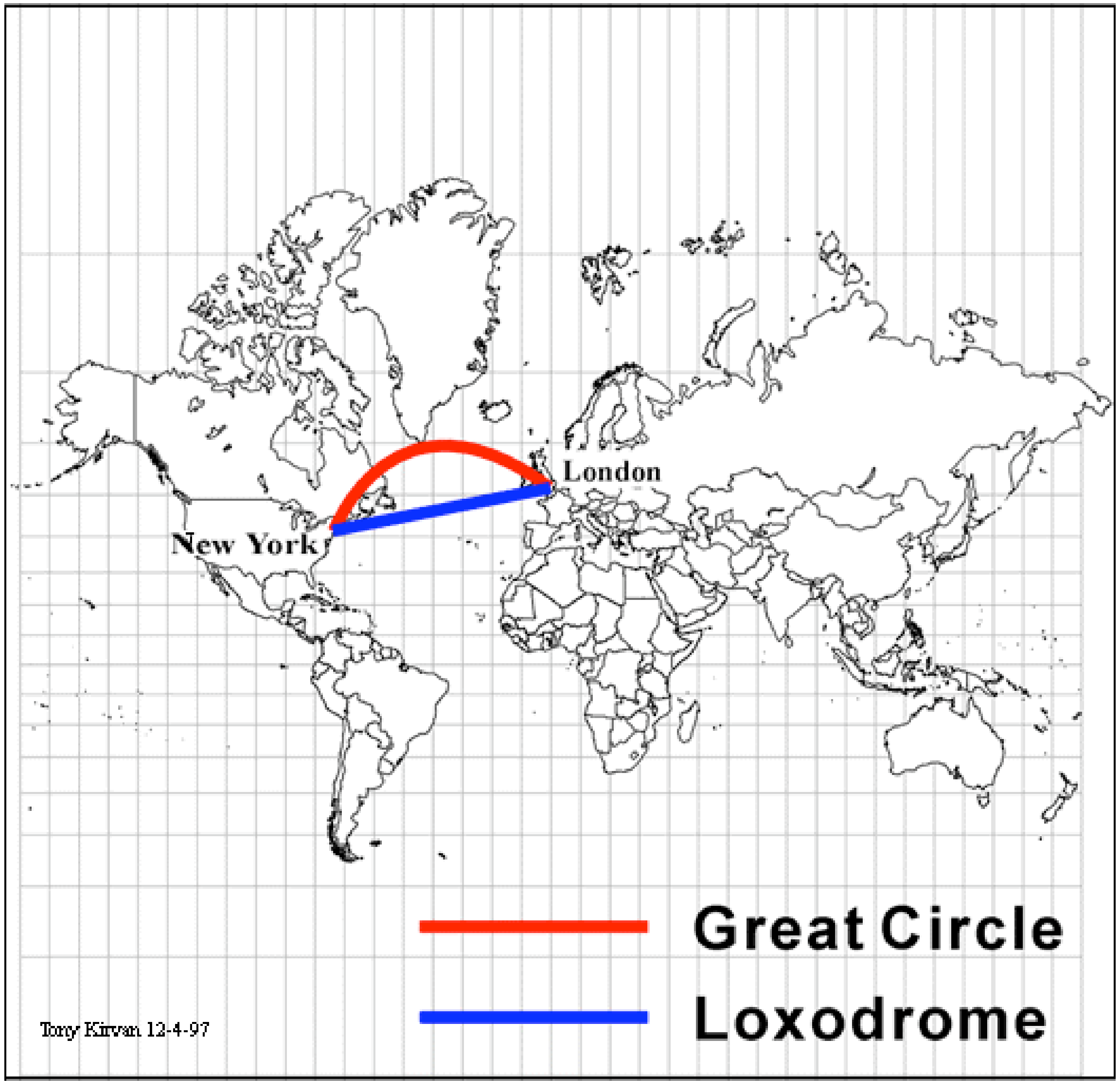




- Conic equidistant projections can be constructed using one or two standard parallels. The simple conic projection uses one standard parallel (tangent case). Any parallel of latitude can be selected as the standard parallel.
- De l'Iles projection is an equidistant conic projection but with two standard parallels.

Great Circle Route and Loxodrome on the Mercator Projection

- The loxodrome is a line of constant heading, and the great circle, although appearing longer than the loxodrome, is actually the shortest route between New York and London.



Homework :-)

- What is the Great Circle Distance in nautical miles between Hamburg (53°37'49"N, 09°59'18"E) and New York (40°38'23"N, 73°46'44"W)?
  - What is the heading (course)
    - when leaving Hamburg (initial heading)?
    - arriving at New York (final heading)?
- I am leaving Hamburg (53°37'49"N, 09°59'18"E) with sog=15 knots, cog=210°. Where am I 40 minutes later (lat, lon)?
  (not a great circle: „Besteckrechnung“)
- Implement a navigation calculator in J2ME which calculates the Great Circle distance in nautical miles, initial heading and final heading for two given coordinates.

This Lecture

- Geoinformatik in Theorie und Praxis
  - Chapter 5.1-5.3
- Great Circles
  - http://plus.maths.org/issue7/features/greatcircles/
- Great Circle Mapper
  - http://gc.kls2.com/

Thank you!

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